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Fraction Instruction for Students with Mathematics Disabilities: Comparing Two Teaching Sequences

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Abstract. This study investigated the effects of teaching middle school students with mathematics disabilities equivalent fraction concepts and procedures using the concrete-representational-abstract (CRA) instructional sequence or the representational-abstract (RA) instructional sequence. Twenty-six students formed the CRA group, and 24 students formed the RA group. The two treatment groups received carefully sequenced instruction over 10 lessons. The only difference between the two treatment groups was that the CRA group used concrete manipulative devices for the first three lessons while the RA group used representational drawings. Analyses of the data indicated that students in both treatment groups improved overall in their understanding of fraction equivalency from pretest to posttest. On all achievement measures, students in the CRA group had overall higher mean scores than did students in the RA group. Implications for classroom instruction and suggestions for further research are discussed.

Historically, students with disabilities have made unacceptably poor progress in mathematics. Wagner (1995) reported that students with disabilities typically perform about two grade levels behind their peers without disabilities, while Cawley and Miller (1989) reported that adolescents with disabilities generally perform at about the fifth-grade level in math. Other researchers have noted that math difficulties emerge in elementary school grades and continue as students progress through secondary school grades (McLeod & Armstrong, 1982; Baroody & Hume, 1991; Engelmann, Carnine, & Steely, 1991; Mercer & Miller, 1992). Specifically, it has been reported that students fail to achieve a sufficient conceptual understanding of the core concepts that underlie operations and algorithms used to solve problems that involve whole and rational numbers (Barody & Hume, 1991; Hiebert, 1986; Hiebert & Behr, 1988). Finally, a number of researchers have discussed concerns related to students' superficial understanding of authentic problem solving (Doyle, 1988; Woodward & Montague, 2002).

Concerns regarding the poor math performance of students with disabilities have increased due to several national trends. First, the 1997 Individuals with Disabilities Education Act (IDEA) amendments emphasize the importance of providing students with disabilities access to the general curriculum. Thus, it is expected that students with and without disabilities will receive instruction on the same important concepts. Second, widespread national acceptance of the National Council of Teachers of Mathematics (NCTM) Standards (2000) has resulted in new challenges for students with disabilities and their teachers. The focus of the NCTM standards is on high-level conceptual understanding and problem solving rather than procedural knowledge and rule-driven computation (Maccini & Gagnon, 2002). These higher-level skills are challenging for many students, but especially for students with disabilities. Finally, many states now require students to pass high-stakes exit examinations as a condition for receiving a standard high school diploma. These examinations

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typically require students to apply mathematic concepts in real-life contexts rather than simply applying rules and procedures to abstract problems. Thus, the mathematical expectations being placed on adolescents with disabilities are becoming increasingly difficult.

Maccini and Hughes (1997) conducted a review of literature related to mathematics instruction for adolescents with learning disabilities. Their review included studies that were published from 1988 to 1995. They found that most of the literature concentrated on instructional methods for teaching facts or rules, with only a few studies dealing with development of conceptual knowledge. Maccini and Hughes noted that, based on this limited research, cognitive and metacognitive strategy instruction and the use of the concrete-representational-abstract (CRA) sequence both were effective for developing conceptual understanding of mathematics. Moreover, these researchers cited the need for additional research to identify effective, validated practices for teaching math to adolescents with learning disabilities.

An area of mathematics that is particularly difficult for students with and without disabilities is understanding fraction concepts (McLeod & Armstrong, 1982; Tourniare & Pulos, 1985). Understanding fraction equivalency is particularly important as it is a fundamental concept underlying the study of ratio, proportion, probability, rates, and functions. According to Hiebert (1985), students have difficulty with fractions because they often fail to connect form and understanding. Hiebert defined form as the syntax, for example, symbols, numerals, and algorithms, while understanding was defined as the ability to relate mathematical ideas to real-world situations. Hiebert identified three specific areas where students should connect form and understanding. First, symbolic representation should connect to real-life cases. An example of this is when students visualize the fraction 3/4 as a pizza with three of four equal-sized pieces remaining. Second, students should connect the algorithm or procedure to the fundamental concept so that they understand why the algorithm or rule works. For example, when fractions with like denominators are added, the denominator does not change. Students should reason that the numerator has increased because we have more pieces, but the denominator has remained constant because we have not changed the size of the pieces. Third, students should connect the answer to a problem with real-life experience. This requires the development of number sense and estimation skills. If we add 3/4 and 11/12, the answer should be close to 2 because both fractions are close to 1 and 1 + 1 equals 2. According to Hiebert, most mathematics curricula and textbooks spend little time on making connections in these areas. He suggested that teachers make much greater use of diagrams and stories to help students visualize fraction concepts.

The National Council of Teachers of Mathematics has articulated national standards related to fraction instruction (NCTM, 2000). The NCTM Standards suggest that all young children (prekindergarten to second grade) should have some experience with simple fractions through connections to everyday events and language (e.g., half). They should understand and represent commonly used fractions such as 1/4, 1/3, and 1/2. According to the NCTM Standards, in grades 3–5 all students should (1) develop understanding of fractions as parts of unit wholes, (2) use models and equivalent forms to judge the size of fractions, and (3) recognize and generate equivalent forms of commonly used fractions, decimals, and percents. In grades 6–8, all students should (1) work flexibly with fractions, decimals, and percents to solve problems, and (2) compare and order fractions, decimals, and percents efficiently. Throughout the NCTM Standards, the importance of providing a variety of fraction models (e.g., fraction strips, grids, objects) and connecting fraction problems to real-life situations is emphasized.

Many authorities have identified components of effective math instruction. However, as noted by Maccini and Hughes (1997), little research focused on math concept development has been done with adolescents with disabilities. Thus, secondary teachers have limited guidance for incorporating the NCTM Standards into math curricula for students with disabilities. This is important when teachers consider including students with disabilities in general education math classes and when they guide students in planning for transition from school to work.

Research supports the use of manipulative devices for teaching a variety of math concepts. Use of both manipulative devices and pictorial representations through the CRA teaching sequence has been effective with algebra (Huntington, 1995; Maccini & Hughes, 2000), basic math facts (Mercer & Miller, 1992), coin sums (Miller, Mercer, & Dillon, 1992), and multiplication (Harris, Miller, & Mercer, 1995; Miller, Harris, Strawser, Jones, & Mercer, 1998; Morin & Miller, 1998; Sigda, 1983). Moreover, studies that involved teaching place value (Peterson, Mercer, & O'Shea, 1988) and algebra (Witzel, 2001) revealed that CRA instruction was more effective than traditional abstract-level instruction. Despite the supportive research for the use of manipulative devices and the NCTM's recommendations for using this type of instruction, there appears to be rather low usage rates among teachers. In a self-report study, Hatfield (1994) found that among teachers who were selected to be cooperating teachers for interns, there was high familiarity with the use of manipulative devices and a high rate of accessibility to the devices. In spite of this, there was limited usage, with a pattern of diminishing use from kindergarten to sixth grade. Undoubtedly, this trend continues into the secondary school grades. There are a number of inherent challenges with using manipulative devices (e.g., managing the dissemination and collection of the devices, providing adequate work space, ensuring that all students have an appropriate number of devices, monitoring accuracy of student performance, managing student behavior). Given these challenges and the apparent preference among upper elementary and middle school teachers to reduce the use of manipulative devices, research is needed to determine whether pictorial representations are just as effective as using concrete manipulative devices along with pictorial representations. This is particularly important with math skills such as fractions because visual representations are critical to understanding basic definitions (e.g., parts of a whole, equivalent parts of a whole). Thus, the present study was conducted to compare the effects of a concrete-representational-abstract (CRA) instructional sequence to a representational-abstract sequence (RA) on the learning of fraction equivalence concepts by middle school students with mild to moderate disabilities.

METHOD

Participants and Setting

The participants in this study were students in a public middle school located in a large urban area of the southwestern United States. Fifty students with mild-moderate disabilities enrolled in grades 6, 7, and 8 formed the two treatment groups. The majority of these students (i.e., 42) had been identified with specific learning disabilities in mathematics. All 50 students received mathematics instruction in resource room settings. The students ranged in age between 11 and 15 years. Subject characteristics including age, gender, grade placement, disability category, and IQ test scores are summarized in Table 1.

TABLE 1
Characteristics of the Two Treatment Groups

	Treatment Group	
	CRA (n = 26)	*RA (n = 24)*
Age		
11	2	2
12	9	10
13	8	8
14	6	4
15	1	0
Gender		
Male	15	12
Female	11	12
Grade Level		
Sixth grade	8	7
Seventh grade	11	12
Eighth grade	7	5
Disability		
Learning disabilities	22	20
Other disabilities	4	4
IQ		
Mean scores	86.19	86.21
Standard deviation	12.772	11.081
Range	56–113	62–109

Note: A *t*-test revealed no significant differences between groups on IQ. Chi-square tests revealed no significant differences on age, gender, grade level, or disability.

An additional 65 eighth-grade students enrolled in general education math classes took the postassessment only. Sixty-three of these students had no disabilities, while two had learning disabilities in areas other than mathematics. These students, 31 boys and 34 girls, ranging in age from 13 to 15 years, formed a comparison group. It is important to note that no attempt was made to control for instructional or other variables. Therefore, the information obtained from these students is used only to give an estimate of what a typical student without disabilities understands about fractions by the end of the eighth-grade year.

Instrumentation

Pretest-Posttest

The pretest-posttest consisted of five subtests and was the primary dependent measure. Three subtests were taken from the Brigance Comprehensive Inventory of Basic Skills-Revised (CIBS-R) (Brigance, 1999), and two subtests were investigator-designed. The CIBS-R is a widely used criterion-referenced assessment specifically designed to be used for instructional placement and program planning in classroom settings. Reliability and validity data are available for its broad math computational and problem-solving placement tests. The available reliability data indicated that the CIBS-R had high degrees of interrater, test-retest, and alternative forms reliability, with excellent internal consistency. The validity data indicated that the CIBS-R had significant correlations with group achievement tests and individually administered diagnostic tests, and that predictive and discriminant validity were high. No data were available on the specific skills subtests. The investigator constructed two subtests to measure skills not assessed in the CIBS-R (Fraction Word Problems and Improper Fractions). The investigator-designed subtests had significant correlations with the subtests taken from the CIBS-R ($p < 0.01$).

Two subtests measured students' knowledge of prerequisite skills, namely, recognition and naming of fractional parts of sets and geometric figures. These two subtests (i.e., Quantity Fractions and Area Fractions) were taken from the CIBS-R (Brigance, 1999). Quantity Fractions consisted of 24 items requiring students to circle the appropriate fractional part of each set and measured students' conceptual understanding of ratio and proportion. For example, students were required to circle 2/3 of 18 dots. Area Fractions had 18 items requiring students to write the proper fraction that was indicated by the shaded portion of a geometric shape and measured part-whole discrimination. The third subtest, Abstract Fractions, also was taken from the CIBS-R. This subtest contained 16 fraction-related items at the abstract level, that is, numbers alone without pictorial representation. The first four items required students to convert a given fraction to a larger equivalent fraction (e.g., 1/3 = ?/12). The next four items required students to convert a given fraction to a smaller equivalent fraction (e.g., 3/6 = ?/2). The final eight items required students to convert mixed numbers to equivalent improper fractions and improper fractions to equivalent mixed numbers (e.g., 8/5 = ? or 2 1/2 = ?/2). The fourth subtest, Word Problems, was designed by the investigator and had 12 problems. It measured the same fraction skills assessed in the Abstract Fractions subtest; however, the fractions were embedded in word problems. For example, "John ate one-third of the pizza, and the pizza had 12 slices. How many slices did John eat?" was a translation of the abstract problem 1/3 = ?/12 described above. The investigator designed the fifth subtest, Improper Fractions, to measure students' conceptual knowledge of mixed numbers and improper fractions since the Area Fractions subtest addressed proper fractions only. This subtest consisted of nine items. Students were required to shade in appropriate parts of geometric shapes or to write a fraction representing shaded-in portions of geometric shapes, that is, when presented with three rectangles each divided into thirds, students were directed to shade in 1 2/3. Since these five subtests had a varying number of items, scores are reported as percentages of correct responses.

Attitude Questionnaire

Students' attitude toward mathematics instruction was measured using an investigator-constructed 10-item questionnaire using a three-point Likert scale (i.e., (1) Don't Agree, (2) OK, or (3) Agree). Raw scores are reported for the attitude questionnaire (note that negatively worded items were reverse-coded before summing.) Thus, a score of 30 indicated agreement on all items, while a score of 10 indicated disagreement on all items (see Table 2).

Materials

Materials for both groups included 10 scripted lessons. Each lesson included an advance organizer, a teacher demonstration, guided practice, independent practice, problem-solving practice, and a feedback routine. Ten student-learning sheets (one per lesson) contained the items for guided practice, independent practice, and problem-solving practice. See Figure 1 for a sample learning sheet. Additionally, four investigator-designed cue cards were used in the study. These cue cards included important definitions and detailed step-by-step procedures for solving fraction problems (see Figures 2–5). Concrete materials included commercially available fraction circles, small white dried beans, and student-made fraction squares of construction paper.

TABLE 2
Attitude Assessment Questions

Directions: Read each question carefully. Circle the number that you think best matches your own feelings.

☹ 1 = I don't agree

😐 2 = OK

☺ 3 = I agree

☹	😐	☺	
1	2	3	Math classes are interesting.
1	2	3	It is easy to get tired of math.
1	2	3	It is fun working on math problems.
1	2	3	Fractions are easy.
1	2	3	I like the math class activities.
1	2	3	Learning fractions is a waste of time.
1	2	3	Math class is dull and boring.
1	2	3	I want to study math in high school.
1	2	3	It is hard to understand fractions.
1	2	3	Knowing math is not helpful when you get out of school.

Procedure

Two special education teachers certified to teach students with mild-moderate disabilities participated in the study. Each teacher taught two sections of math per day. One of each teacher's classes was randomly designated the CRA group, while the other was designated the RA group. Thus, the sample consisted of two CRA classes and two RA classes. This counterbalancing was used to control for possible teacher effects. A total of four phases were used in this study: teacher training, preassessment, intervention implementation, and postassessment.

Phase 1: Teacher Training

The investigator conducted a three-hour training session for the teachers involved in the study. The purpose of the training session was to promote consistency among the groups and to minimize any possible teacher effects. The session began with an overview of the instructional unit, including the lesson objectives, teaching methodology, and lesson formats. Then, each teacher taught a 30-minute demonstration lesson while being observed. Specifically, teachers were observed for the following five criteria: (1) using explanations that were consistent with the strategies, (2) providing appropriate and adequate student feedback, (3) monitoring students during guided and independent practice, (4) following the sequential order of the lesson, and (5) allowing adequate time for each activity. To

1) Circle 1/3 of the 3 ■ ■ ■ ■	2) Circle 3/4 of the 8 ■ ■ ■ ■ ■ ■ ■ ■ ■ 3/4 =	3) Circle 2/5 of the 10 ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ 2/5 =	4) Circle 5/6 of the 18 ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ 5/6 =
5) Circle 3/5 of the 20 ■ 3/5 =	6) Circle 7/12 of the 24 ■ 7/12 =	7) Circle 2/6 of the12 ■ 2/6 =	8) Circle 7/8 of the 32 ■ 7/8 =
9) Circle 2/3 of the 9 ■ 2/3 =	10) Circle 3/8 of the 16 ■ 3/8 =	11) Circle 9/10 of the 20 ■ 9/10 =	12) Circle 1/4 of the 12 ■ 1/4 =

13) Jose had 12 sports trading cards. 1/3 of the cards were football cards. How many cards were football cards? Write your answer as a fraction.

14) In our reading class, 5/6 of the students are boys. There are 18 students altogether. How many students are boys? Write your answer as a fraction.

FIGURE 1 Learning sheet #2.

Fraction Words

- Fraction bar—the line that divides the numerator from the denominator
- Numerator—the number above the fraction bar; tells us how many parts of the whole unit are shaded or circled
- Denominator—the number below the fraction bar; tells us how many equal parts make up one whole unit
- Proper fraction—a fraction in which the numerator is less than the denominator; represents less than one whole unit
- Improper fraction—a fraction in which the numerator is equal to or greater than the denominator; represents one whole unit or more and can be converted to an equivalent mixed number
- Mixed number—a number made up of a whole number and a proper fraction; can be converted to an equivalent improper fraction
- Equivalent fraction—a fraction that has the same value as another fraction

FIGURE 2 Student cue card: fraction word definitions.

obtain an estimate of fidelity of treatment, two observers (i.e., Learning Strategy Specialist and Assistant Principal) independently rated the demonstration lessons. The observers sat in the back of the classroom while they completed a checklist consisting of the five criteria. After the demonstration lesson, the observers compared checklists. There was 100 percent interobserver agreement that both teachers performed all five items adequately. This feedback was given to each teacher at the conclusion of the training session.

During the study, two additional observations of each teacher were held, once after Lesson 2 and once after Lesson 7. Feedback was given to each teacher according to the criteria listed above. Again, 100 percent interobserver agreement was reached in each of these two observations. The first observation resulted in one teacher being directed to include more feedback and practice at the conclusion of the lesson, while the second observation noted that both teachers performed all five criteria adequately.

Fundamental Law of Fractions

The value of a fraction does not change if its numerator and denominator are multiplied by the same number.

This is true because the value of a number does not change when it is multiplied by one.

Examples:

- 1/1, 2/2, 3/3, 5/5, and 10/10 are all different names for the number one. There are an infinite number of different names for one.
- 1/2, 2/4, 3/6, 5/10, and 10/20 are all equivalent fractions. For any fraction, there are an infinite number of equivalent fractions.
- To write an equivalent fraction, choose a fraction for the number one. Multiply by that fraction.

  $\frac{2 \times 5 = 10}{3 \times 5 = 15}$ so, 2/3 = 10/15

FIGURE 3 Student cue card: fundamental law of fractions and examples.

Reducing Fractions to Simplest Terms

Step 1: Find the GCF of the numerator and denominator.

Step 2: Divide the fraction by a fraction equivalent to 1 that has the GCF as the numerator and denominator.

This is true because the value of a number does not change when it is divided by 1.

Example:

Write 9/21 in simplest terms.

1. Find the GCF of 9 and 21.
   - The factors of 9 are 9, 3, and 1
   - The factors of 21 are 21, 7, 3, and 1
   - The GCF of 9 and 21 is 3.
2. Divide 9/21 by 3/3.
   - 9 ÷ 3 = 3
   - 21 ÷ 3 = 7
3. The fraction 3/7 is in simplest terms.

FIGURE 4 Student cue card: reducing fractions to simplest terms and example.

Phase 2: Preassessment

The pretest-posttest, consisting of five subtests designed to measure students' understanding of fraction concepts, procedures, and application, was administered to all subjects during one 50-minute regularly scheduled mathematics period. Students were encouraged to try their best, and directions were reread or restated as necessary. No other prompting was given. Students worked independently. The attitude questionnaire was administered on the next day immediately before beginning Lesson 1.

Phase 3: Intervention Implementation

Each daily lesson followed a predictable format based on the methodology of the *Strategic Math Series* (a program designed for teaching math facts and place value) (Mercer & Miller, 1991–1994). This program was used as a model for developing the fraction lessons used in this study because it uses the CRA instructional sequence and has been validated with students who have learning difficulties in math. Teachers used scripted lessons and accompanying learning sheets to progress through each of the following seven components.

- *Advance Organizer*—The teacher linked the current lesson to previous instruction, identified the daily objective, and gave a rationale for learning the skill.
- *Teacher Demonstration*—The teacher first demonstrated the skill while describing aloud the steps; then the teacher and students solved two problems together through a question-and-answer format.
- *Guided Practice*—The teacher gave prompts and cues as students solved three problems together. As students gained independence, the teacher monitored students and assisted only as needed.
- *Independent Practice*—Students solved seven problems independently using the skills that had been taught. The teacher did not provide assistance.
- *Problem-Solving Practice*—The teacher and students read the first word problem together, and students applied the skills they had been taught in the lesson. Then students completed the second word problem independently.
- *Feedback Routine*—Learning sheets were collected and scored daily, and students who achieved a score of 80 percent or better were deemed to be at criterion level for that day's lesson. The teacher met individually with each student to discuss the daily work and give corrective feedback. In only two cases did students not achieve criterion. One student in the CRA group did not meet criterion in Lesson 2, and one student in the RA group did not meet criterion in Lesson 3. They were directed to rework the problems under teacher guidance, and in both cases criterion was met that same day. It is important to note that during the corrective procedure, the student in the CRA group was allowed to use concrete manipulative devices, while

Changing Improper Fractions to Mixed Numbers

Step 1: In any fraction, the fraction bar that separates the numerator from the denominator means "divide." To express an improper fraction as a mixed number, divide the numerator by the denominator.

Step 2: List the remainder as a fraction of the divisor.

Examples:

Express 15/2 as a mixed number.

1. 15 ÷ 2 = 7 r 1
2. 15/2 = 7 1/2

Express 45/5 as a mixed number.

1. 45 ÷ 5 = 9 (there is no remainder)
2. 45/5 = 9

FIGURE 5 Student cue card: changing improper fractions to mixed numbers and examples.

the student in the RA group used only pictorial representations.

- *Cue Cards and Notes*—Students in both treatment groups were provided printed notes and cue cards developed by the investigator in case they became confused or forgot the steps for solving a problem. Students placed the cue cards and notes in their individual notebooks for easy access in class.

Lesson Sequence

For the CRA group, the first three lessons were designed to introduce the concept of fraction equivalence through the use of concrete manipulative devices. For example, in Lesson 1, students used commercially available fraction circles to explore fraction equivalency and nonequivalency. Lesson 1 presented similar tasks to those in the Area Fractions subtest. In Lesson 2, this concept was extended to proportional reasoning through the use of small white beans. The tasks presented in Lesson 2 were similar to those in the Quantity Fractions subtest. Specifically, in a problem such as 3/4 = ?/16, students were taught to count beans representing the denominator of the equivalent fraction (i.e., 16). Next, students separated that set into the number of groups called for in the denominator of the given fraction (i.e., 4). Finally, students looked at the numerator of the given fraction to decide how many groups to combine to form the numerator of the equivalent fraction (i.e., 3 groups of 4 each, or 12). Lesson 3 was a continuation and review of fraction equivalence using folded construction paper as the manipulative device in order to promote generalization to various types of concrete objects. Also in Lesson 3, students were taught to express mixed numbers as improper fractions.

Concept development continued in Lessons 4–6. These lessons involved the use of representational drawings to represent fraction equivalence and were similar to items in the Improper Fractions, Area Fractions, and Quantity Fractions subtests. In Lesson 4, students learned to express improper fractions as mixed numbers. For example, in the fraction 39/12, students subdivided large rectangles into 12 equal parts. Then, they shaded in 39 parts that represented 3 whole rectangles and 3/12. In Lesson 5 students converted mixed numbers into improper fractions. Lesson 6 was a continuation and review of all of the previous lessons, including proper fractions, improper fractions, and mixed numbers.

In Lesson 7, students were introduced to the abstract algorithm for computing equivalent fractions. In Lessons 8–10, students were provided practice in applying the algorithm to abstract problems with proper fractions, improper fractions, and mixed numbers. These problems were similar to those presented in the Abstract Fractions subtest.

In each of the daily lessons, beginning with Lesson 1, students learned to solve word problems containing fractions. Specifically, students learned to apply the procedures learned in that day's lesson to discover the solution to word problems. These items were similar to the problems in the Word Problems subtest.

Students in the RA group received the same amount of instructional time (i.e., 10 45-minute lessons), with each lesson following the same instructional sequence as students in the CRA group. However, instead of using manipulative devices during Lessons 1–3, students in the RA group used representational drawings demonstrating the concepts being taught. For example, in Lesson 1, students in the RA group used drawings of circles and divided them into equal fractional parts to represent proper fractions, whereas students in the CRA group used commercially available fraction circles. Similarly, in Lesson 2, the CRA group used small white beans to repartition sets; students in the RA group performed the same tasks, but used drawings of small squares arranged in arrays instead of beans. They were taught to lightly draw the subgroups, then circle darkly the number of groups called for in the numerator of the fraction (see Figure 1). This was difficult for some students with minor motor problems. Lesson 3 was a continuation and review of fraction equivalence using drawings of squares that students shaded in to represent various fractions that were written in number format on the learning sheet. In this lesson, students also were taught to express mixed numbers as improper fractions. Lessons 4–10 were the same for both the RA and CRA groups.

Phase 4: Postassessment

The postassessment consisted of the same five subtests and attitude questionnaire used in the preassessment and was administered to the two treatment groups after all 10 scripted lessons were completed.

The pretest-posttest and attitude questionnaires were also administered to a comparison group consisting of 65 eighth-grade students who were enrolled in three general education math classes designed for students of average ability levels. Six months prior to this study, these students had taken the Terra-Nova Comprehensive Test of Basic Skills, 5th ed. (CTB/McGraw-Hill, 1996–1997). When compared to a national group of eighth-grade students, they achieved a mean stanine score of 4.75 on the mathematics portion, indicating math ability in the average range.

The students in the comparison group had just completed approximately three weeks' instruction on rational numbers and ratio and proportion. The instructor, who was licensed by the state to teach secondary mathematics, followed the district-prescribed curriculum for eighth-grade mathematics using a district-adopted basal text (i.e., *Math Applications and Connections*). Two of the students in the general education comparison group were identified with specific learning disabilities, but their areas of deficit did not include mathematics, and they received all of their math instruction in the general education classroom without modifications or adaptations. Thus, postassessment data were collected on 115 students (i.e., 65 general education and 50 special education).

Interrater Reliability

To obtain an estimate of scoring reliability, two raters independently scored a random sample of 20 percent of the tests that were administered in this study. Using the formula (agreements $\div$ {agreements + disagreements} $\times$ 100), interrater agreement was calculated to be 97 percent.

RESULTS

Pretest-Posttest Fraction Subtest Comparisons

The Area Fractions, Quantity Fractions, and Improper Fractions subtests provided a measure of students' conceptual understanding of fraction equivalency, while the Abstract Fractions and Word Problems subtests measured students' ability to apply the algorithms for solving fraction equivalencies. Paired samples *t*-tests were used to test for differences between the pretest and posttest measures for the CRA and RA groups. Results of the *t*-tests for the fraction subtests showed significant improvement ($p < 0.05$) on all measures for both groups except for the CRA group's difference on the Area Fractions subtest. Effect sizes for the significant findings were estimated using Cohen's d. The Cohen's d values ranged from small for the Area Fractions difference for the RA group to large and very large for the remaining differences on the fraction subtests. These data, along with pretest and posttest means and standard deviations, are presented in Table 3.

Differential Gain in the CRA and RA Interventions

This study was specifically designed to investigate potential differences between the CRA and RA treatments on the fraction measures. Treating pretest scores as covariates, a MANCOVA was used to test for treatment effects for the set of five posttest fraction subtests. The difference between the two treatment groups was statistically significant for the set of five fraction measures ($F_{(5,49)} = 2.811$, $p = 0.029$, using Wilk's criterion, eta squared was 0.265). Follow-up univariate tests revealed a statistically significant difference for only the Quantity Fractions subtest, which favored the CRA group ($F_{(1,43)} = 14.759$, $p < 0.0005$), although both the raw and adjusted posttest means were higher for the CRA treatment on all five meaasures.

Fraction Mastery of the Target Students

The two treatment groups were combined for a comparison of mastery levels with the traditional group. Table 4 reports means and standard deviations for the subtest fraction posttests for the combined treatment groups and the traditional group. The MANOVA comparing treatment subjects with the traditional group revealed a statistically significant difference for the set of dependent variables between the groups ($F_{(5,109)} = 9.003$, $p < 0.0005$, using Wilk's criterion). Eta squared was 0.292, indicating a moderate association between the independent variable and the set of dependent variables. Univariate follow-up tests of between-subjects effects found significant differences between

TABLE 3
Pretest-Posttest Comparison for Treatment Groups

	Pretest		Posttest			
Measure	*Mean*	*SD*	*Mean*	*SD*	*t-value*	*Cohen's d*
CRA ($n = 26$)						
Attitude	20.81	5.15	23.42	2.30	2.768*	1.11
Area Fractions	88.42	24.36	97.81	3.66	1.912	0.08
Quantity Fractions	30.54	23.83	77.31	20.63	7.267*	2.91
Abstract Fractions	16.08	22.46	79.46	21.54	11.963*	4.79
Improper Fractions	15.31	22.44	70.96	18.69	10.252*	4.10
Word Problems	6.69	19.37	69.12	30.68	9.382*	3.75
RA ($n = 24$)						
Attitude	21.92	4.14	24.21	3.28	2.277*	0.95
Area Fractions	80.46	35.84	97.33	3.75	2.264*	0.94
Quantity Fractions	28.42	17.20	55.33	21.95	6.682*	2.79
Abstract Fractions	8.21	9.56	70.54	32.59	9.291*	3.87
Improper Fractions	18.79	19.05	63.42	28.14	8.387*	3.50
Word Problems	3.46	9.52	63.42	37.30	7.679*	3.20

Note: The values represent mean percentages of correct responses.
*Significant differences are starred ($p < 0.05$).

the groups favoring the treatment group on the Improper Fraction subtest ($F_{(1,113)} = 18.642$, $p < 0.0005$) and on the Word Problem subtest ($F_{(1,113)} = 6.833$, $p = 0.010$). The alpha level for the univariate tests was corrected according to Bonferroni's correction, yielding an adjusted α of 0.010. While significance testing for nonequivalent groups should be interpreted with caution, the results support the observation that the treatment students mastered much of the material at least as well as the general education comparison students.

Attitude Measure Comparisons

Using the attitude pretest as a covariate, an ANCOVA comparing posttest attitudes between the CRA and RA groups found no significant difference between the groups.

Paired samples *t*-tests were used to test the differences between the pretest and posttest attitude measures for the CRA and RA groups (see Table 3). Each response to the 10-item attitude survey was coded, with 1 being a negative response, 2 a neutral response, and 3 a positive response, for a total possible of 30 points. Both groups were near to a neutral attitude on the pretest, with means 20.81 for the CRA group and 21.92 for the RA group. Although *t*-tests revealed a statistically significant difference between pretest and posttest attitudes, the gains were large. The CRA group had a mean difference of 2.62, ($t_{(25)} = 2.768$, $p = 0.01$, Cohen's d = 1.11), while the RA group had a mean difference of 2.29, ($t_{(23)} = 2.277$, $p = 0.03$, Cohen's d = 0.95).

TABLE 4
Posttest Comparison of Combined Treatment and Traditional Groups

	Treatment ($n = 50$)		Traditional ($n = 65$)		
Measure	*Mean*	*SD*	*Mean*	*SD*	*F-value*
Area Fractions	97.58	3.671	93.52	18.359	2.364
Quantity Fractions	66.76	23.794	65.62	30.451	0.48
Abstract Fractions	75.18	27.487	83.80	23.155	3.326
Improper Fractions	67.34	23.760	44.94	30.183	18.642**
Word Problems	66.38	33.786	50.18	32.269	6.833*

Note: The values represent mean percentages of correct responses.
**$p < 0.0005$, *$p = 0.010$.

DISCUSSION

Pretest-Posttest Subtest Comparisons

The purpose of this study was to investigate the effects of the CRA and RA instructional sequences on the learning of equivalent fraction concepts and procedures by students with mild to moderate disabilities. Students in both treatment groups improved significantly in achievement after the 10-lesson intervention. As estimated by Cohen's d (see Table 3), the intervention was judged to be practically significant as well on all measures except Area Fractions. For Area Fractions it should be noted that pretest mean scores were above 80 percent, while posttest mean scores were over 97 percent. An examination of the pretests revealed that most students could recognize and name fractional parts of geometric figures, indicated by the high scores on the Area Fractions pretest. However, they had no clear understanding of equivalencies, abstract problems, or word problems. In many cases, these problems were not even attempted on the pretests. After instruction students were able to decide on appropriate methods of solving problems, and they often drew graphic representations. All the students in both treatment groups attempted all the posttest questions.

Differential Gain in the CRA and RA Interventions

Post hoc tests revealed a significant difference in the treatment groups only in Quantity Fractions. Thus, the CRA group demonstrated better conceptual understanding of fraction equivalency than did the RA group. Although not statistically significant, the mean scores for the CRA group on all subtests were higher than were those of the RA group. This finding is interesting because the only difference between the two treatment groups was the use of concrete manipulative devices for the initial three lessons in the CRA group. A future study might determine if the use of manipulative devices for a longer period of time would produce significant differences in the other skills assessed.

Fraction Mastery of the Target Students

Students in the two treatment groups performed at least as well as did students in the comparison group of general education eighth graders on all the fraction subtests. This is important when we consider the implications for access to the general curriculum and performance on high-stakes assessments. The present study illustrates one method by which teachers can help level the playing field for students with disabilities.

In addition, students in the two treatment groups had significantly higher scores on improper fraction-mixed number conversion when this task was presented graphically on the Improper Fraction subtest. Interestingly, there were no significant differences among the groups when the identical items were presented numerically on the Abstract Fractions subtest. An examination of students' papers revealed that many students in the comparison group did not connect the denominator of a fraction to the number of parts of each whole unit. In one test item, students were asked to shade 6/3 rectangles. The item contained two rectangles divided into 6 equal parts each. In a typical incorrect response (seen only in the comparison group), one whole rectangle was shaded, or 6/6. Students appeared to understand that the numerator should indicate the number of parts shaded, but they failed to connect the denominator, 3, with the number of parts each whole should contain. And, even though the comparison students knew how to convert improper fractions to mixed numbers on the Abstract Fractions subtest, they did not apply the algorithm to derive the answer $6/3 = 2$. Logically, two whole rectangles should be shaded, regardless of the number of partitions in each rectangle. This type of error may be due to the instructional method used in the traditional curriculum. Although students were exposed to representative diagrams in their basal textbook, they were not taught systematically how to draw their own representations as an aid to analyzing problems. Moreover, when fraction concepts were taught, students were given illustrations of proper fractions, and few of the practice problems involved improper fractions or mixed numbers.

Finally, the treatment groups had significantly higher scores than did the comparison group on solving word problems with embedded fraction equivalencies. This finding is interesting because the problems presented were identical to those presented in the Abstract Fractions subtest, and there were no significant differences among groups for that subtest. Whereas the comparison group had the highest mean scores on the Abstract Fractions subtest, they had the lowest mean scores for the Word Problem subtest. An examination of individual student papers showed that some students in the treatment groups solved the word problems using graphic representations, while students in the comparison group appeared to rely on application of an algorithm. The use of the graphic representations may have enabled students who had forgotten the algorithm to solve the problem by reasoning it out with a drawing. As previously noted, treatment group students were explicitly taught how to draw representational drawings, while comparison group students' instruction focused on application of algorithms. Even though comparison group students knew how to solve the same problems when they were presented abstractly, they appeared not to transfer that knowledge to the graphic representations.

Attitude Measure Comparisons

For the most part, students in the treatment groups had neutral to slightly positive attitudes toward mathematics and fractions, and there were no differences between the CRA and RA groups. This finding has two implications. First, students who use manipulative devices do not seem to feel more babyish or less competent than those who do not, and second, they do not seem to feel that math is more fun than their peers who do not use manipulative devices. This finding may help teachers who hesitate to use hands-on, manipulative devices with adolescents.

Although the pretest-posttest attitude comparison revealed significant gains, it is difficult to draw conclusions given the short duration of this intervention. It should be noted that these students had a long history of struggling with mathematics. Although any improvement in attitude is important, it is probable that mathematics will continue to be a challenge for most of these students.

Study Limitations

Several limitations encountered in the process of implementing this study should be considered when interpreting these data. This study was conducted at the end of the school year. Due to a shortage of time, the study was limited to 10 lessons, and no maintenance probes were performed. Thus, follow-up data on generalization and maintenance were not available. Most of the students in the treatment groups were identified with specific learning disabilities, although a few students had attention deficit disorder, emotional disabilities, or mild mental retardation. Therefore, caution should be used in generalizing the data to students with disabilities other than learning disabilities. The results of this study were obtained with group instruction in a resource room setting. Students spanned three grade levels, and placement was determined by deficits in math rather than by diagnostic category or grade level. This study did not address instruction of students with disabilities in general education settings in which placement is determined by grade level regardless of math ability.

Practical Implications

Both the CRA and RA instructional sequences are easily implemented in middle school classroom settings with students who have mathematics disabilities. The needed materials can be obtained at low cost or can be made by the teacher or students. In addition, several validated instructional variables were included in the lessons for the two treatment groups. These included careful sequencing of skills, selection of appropriate examples, immediate feedback, and mastery learning.

A task analysis of the required skills was needed so that the lessons could be properly sequenced. This information is available in the scope and sequence sections of most math textbooks. For this study, information found in *Designing Effective Mathematics Instruction: A Direct Instruction Approach* was used (Stein, Silbert, & Carnine, 1997). Each lesson was developed with an appropriate range of examples, adequate practice, and clear explanations (Baroody & Hume, 1991; Brigham, Wilson, Jones, & Moisio, 1996; Maccini & Hughes, 1997).

Ongoing assessment was an important part of this instructional sequence because it allowed the teachers to make changes as needed in direct response to the needs of the learners. In two cases, students failed to reach the 80 percent criterion in a single lesson. When this happened, the teacher and student met individually to discuss the errors. Then, the student completed another guided practice problem while verbally explaining the steps as he worked. In these two cases, the guided practice was successful, and each student corrected the errors on the learning sheet to achieve the 80 percent mastery criterion.

Suggestions for Further Research

Future research is needed to determine the stability of these results over time. The skills that were taught in this intervention are used in operations with fractions and in computing ratio and proportion problems. Further research is needed to determine whether students would be able to draw representations to solve problems even if they forgot the abstract algorithms.

This study involved students with disabilities who received their math instruction in resource rooms containing students of differing ages and grade levels. Future research would be useful in determining the applicability of the CRA instructional sequence for students with disabilities in general education classrooms who receive instruction with their same-grade peers. Such a study could also address the applicability of this instructional method to students without disabilities.

Finally, this study yielded information relative to the acquisition of concepts of fraction equivalency. Future research would be useful in exploring how the CRA method incorporating validated instructional design principles might be used in teaching operations with fractions or algebra concepts. Such a study might compare students who participated in CRA instruction for fraction equivalency to students who had traditional instruction in fraction equivalency. Both groups would be given CRA instruction for the new concepts. This could help determine the role of conceptual understanding in developing higher-level mathematics skills.

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