

# Effects of Manipulative Instruction on Solving Area and Perimeter Problems by Students with Learning Disabilities

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*Abstract.* A multiple baseline design was employed to test the effect of manipulative instruction on the perimeter and area problem-solving performance of middle and high school students who had been diagnosed with LD in the area of mathematics. Modeling, prompting/guided practice, and independent practice in conjunction with manipulative training were employed to teach both perimeter and area problem-solving skills. Analysis of data revealed that the students rapidly acquired the problem-solving skills, maintained these skills over a two-month period, and transferred these skills to a paper and pencil problem-solving format. This research extends previous findings by revealing that use of concrete manipulatives promotes the long-term maintenance of skills.

Students who wish to function successfully as adults in the 21st century must acquire an understanding of mathematical concepts, learn to reason and problem-solve, and develop a positive attitude toward mathematics (National Council of Teachers of Mathematics (NCTM), 2000). It is crucial for students to realize that math is an integral part of everyday life rather than just a series of problems to be solved in a textbook. Unfortunately, many students have a difficult time acquiring and employing math skills. Lerner (2003) notes that between 6 percent and 7 percent of students in general education evidence an arithmetic disability. Analysis of data from the Third International Mathematics and Science Study (National Center for Education Statistics, 1996) reveals that eighth-grade and 12th-grade students from the United States scored below the international average in math. Further examination of the data indicated that students in the United States practiced routine

procedures during 96 percent of their seatwork time. This situation might, in part, account for the below-average ranking of students from the United States.

One group of adolescents who have a particularly difficult time acquiring and applying mathematical skills are those evidencing learning disabilities (LD). Approximately 26 percent of students labeled as having LD exhibit learning problems in the area of mathematics (Brian, Bay, Lopez-Reyna, & Donahue, 1991). More than 50 percent of students evidencing LD have math goals included in their individual education plans (Kavale & Reese, 1992). A disability in the area of math not only presents problems during the school years but also can interfere with functioning throughout adult life (Miller & Mercer, 1997; Patton, Cronin, Bassett, & Koppel, 1997; Shalev, Manor, Auerbach, & Grodd-Tour, 1998). Adolescents manifesting LD exhibit difficulties in several areas of math, including basic skills (Algozzine, O'Shea, Crews, & Stoddard, 1987), problem application (Cawley & Miller, 1989), and grasping higher-level skills and problem solving (Huntington, 1994; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Witzel, 2001). Given the difficulties encountered by adolescents with LD in learning math skills via traditional instruction involving mainly abstract representation of problems and their solutions, it is imperative that instructional strategies be identified that facilitate the acquisition, maintenance, and generalization of math skills. One way to accomplish this task is through the inclusion of manipulative instruction; however, research in this area of instruction has been deficient.

## PREVIOUS RESEARCH ON THE VALUE OF MANIPULATIVE MATERIALS

Manipulative materials are concrete materials such as geoboards, pattern blocks, chip-trading boards, counters, algebra tiles, attribute pieces, fraction bars, and

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Cuisenaire rods that students arrange in some way to represent a variety of mathematical relationships (Maccini & Gagnon, 2000). A perusal of the literature revealed that only a few empirically-based studies have evaluated the effect of using manipulative instruction on the acquisition and retention of math concepts by students with LD. These include several studies in each of the following areas: (1) computation/place value, (2) word problems, and (3) fractions.

### Computation/Place Value Studies

Peterson, Mercer, and O'Shea (1988) employed a concrete-semiconcrete-abstract (CSA) teaching sequence in conjunction with direct instruction to teach three elementary-level male students with LD place value skills. Concrete instruction materials included plastic cubes, teacher-made place value strips, and place value cards. Pictorial representations of place value sticks and cubes were used during semiconcrete instruction, and no representations were used during the abstract phase of instruction. Immediate and delayed posttests revealed that students had mastered place value identification with an 80 percent or greater level of accuracy.

Funkhouser (1995) taught basic number concepts to K-1 students evidencing LD by employing a manipulative device. Students were presented with a vertical display of a rectangle divided into five equal squares, with dots or jellybeans placed within the squares in order to represent the numbers 0 through 5. During the course of instruction, the students constructed identical configurations that represented the numbers 0 through 5, combined configurations to discover sums between 0 and 5, and used the + sign to create sums between 0 and 5. At the end of the four-week intervention period, all students displayed a 90 percent or greater level of mastery in recognizing and matching the numbers 0 through 5 and adding sums up to 5.

Miller, Mercer, and Dillon (1992) taught 15 students attending elementary school (10 classified as having LD, 3 classified as at risk for LD, 1 classified as mentally retarded, 1 classified as emotionally disturbed) basic division, addition, and subtraction facts, place value, and coin sums to 50 cents using a CSA teaching sequence and a four-step lesson format. The four steps included providing an advanced organizer, demonstrating the desired skill for the student to model, providing guided practice with feedback, and allowing the student time for independent practice. Data analysis indicated that all students demonstrated 80 percent or greater mastery of the skill they were taught.

Mercer and Miller (1992) evaluated the CSA instructional sequence when field testing their Strategic Math Series. A total of 22 teachers provided instruction to 109 elementary students, 102 of whom were identified as having LD. The same four-part lesson format noted above was employed to teach three concrete, three semiconcrete, and three abstract lessons. Posttest and

retention measures revealed the students were able to solve and create word problems, increase their rate of computation, acquire basic computation skills across facts, and generalize skills to other people, tasks, and loci.

Miller, Harris, Strawser, Jones, and Mercer (1998) employed a CSA teaching sequence in combination with modeling, demonstration, feedback, and cognitive training to teach basic multiplication skills to students with disabilities, students achieving at a low level, and students achieving at a normal level enrolled in inclusive second-grade classes. During the concrete phase, plates and plastic dies were used to represent and solve problems. During the semiconcrete phase, students used drawings and tallies to solve problems. During the abstract phase, students simply used the symbols to solve the problems. Data analysis indicated that all students, including those with LD, profited from instruction in inclusive settings with 25 to 27 other students.

### Word Problems

Marsh and Cooke (1996) evaluated the effect of employing manipulatives (Cuisenaire rods) to teach three third graders with LD to solve simple word problems. Data analysis indicated that the students were not only more successful in identifying the correct operation to employ when using manipulatives, but also generalized their newly learned problem-solving skills to abstract situations. Marsh and Cooke noted that their findings called into question the need for including the semiconcrete portion of instruction in the CSA teaching sequence.

Huntington (1994), Maccini and Hughes (2000), and Maccini and Ruhl (2000) evaluated the effect of a CSA teaching sequence on the algebra word-problem-solving skills of three, six, and three adolescent students with LD, respectively. During the concrete phase, students used algebra tiles to represent relational statements. Once this skill was mastered, they represented problems via pictorial arrangement, and upon mastery at this stage they then employed mathematical symbols to represent and solve problems. Students in Huntington's study were exposed to teacher modeling and guided practice, and were allowed time for independent practice. Maccini and Hughes (2000) and Maccini and Ruhl (2000) employed the STAR strategy (search the word problem, represent the problem, answer the problem, review the answer) to teach their students to solve integer algebra word problems. Students in each study improved in their ability to represent and solve word problems.

### Fractions

Jordan, Miller, and Mercer (1998) compared the teaching of fraction concepts and skills to fourth-grade students by means of a CSA instructional sequence to teaching by means of traditional textbook instruction.

Of the 125 students participating in the study, only five were classified as having LD. During the concrete instruction phase, students used laminated construction paper circles cut into fraction strips and pieces. During the semiconcrete phase, students used worksheets with fraction representations and fraction reference cards. Students in the comparison groups completed work assigned by their teacher based on the textbook material. Data analysis revealed that students who received the CSA instruction evidenced greater gains in fraction concepts and skills than did students receiving traditional instruction.

### Summary and Needs for Additional Research

Evaluation of the use of manipulative materials with students classified as having LD revealed that manipulative instruction was effective in teaching place value skills (Peterson et al., 1988), word-problem-solving skills (Huntington, 1994; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Marsh & Cooke, 1996; Witzel, 2001), computation skills (Funkhouser, 1995; Mercer & Miller, 1992; Miller et al., 1998; Miller et al., 1992), and fraction-solving skills (Jordan et al., 1998). The majority of researchers employed a CSA teaching sequence in combination with some type of direct instruction that typically included modeling, demonstration, and guided practice with feedback. Elementary school students were employed as participants in most studies. Only four researchers investigated the effect of manipulative training on adolescents with LD (Huntington, 1994; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Marsh & Cooke, 1996; Witzel, 2001). Many of the investigators assessed the maintenance of skills acquired following training; however, these measures were typically obtained only two to four weeks after instruction ceased.

Thus, a study of the impact of manipulative instruction on students with LD in the area of geometry is warranted for several reasons. No studies were found that investigated the impact of manipulative instruction on the area and perimeter problem-solving skills of adolescents with LD. In addition, only one group of researchers examined the effect of employing just the concrete phase of instruction on acquisition, maintenance, and generalization of skills to an abstract level (Marsh & Cooke, 1996). Also, there is a need to assess long-term maintenance of skills acquired. Moreover, a study in this area is justified because effective means to address NCTM Standards in the area of geometry as well as measurement are needed. Specifically, the geometry standard (NCTM, 2000) calls for "geometry to be learned using concrete models, drawings, and dynamic software." The measurement standard requires students to apply appropriate techniques, tools, and formulas to determine measurements. Further, mastery of skills noted in this standard is considered crucial because measurement is employed in many aspects of everyday life.

### PURPOSE/STUDY QUESTIONS

The purpose of this study was to evaluate the effect of manipulative instruction on the acquisition and maintenance of perimeter and area problem-solving skills by students with LD in math. Specifically, the following questions were addressed: (1) How long would the students with LD in math take to acquire skills in perimeter and area problem solving? (2) Once the skills in area and perimeter problem solving were mastered, would the skills be maintained over time? and (3) Would the students be able to transfer the area and perimeter problem-solving skills learned through manipulatives to paper-pencil calculations as well as real-life problem solving?

### METHODOLOGY

#### Participants

Three students classified as LD were selected from a junior/high school located in a rural area of the central United States. Of the approximately 100 students served by this school, 50 percent are African American, 40 percent are Caucasian, 5 percent are Native American, and 5 percent are Hispanic. The average class is comprised of 15 students. Approximately 50 percent of the students come from single-parent families. The SES status of most families is in the below-average range, and the majority of the family wage earners are employed in either agriculture or heavy industry manufacturing jobs.

Student criteria for participation in the study included: (1) a significant discrepancy between intelligence and mathematical achievement, (2) an area and perimeter geometry pretest score of less than 40 percent, (3) lack of any formal training in any area or perimeter concepts, and (4) identification by the resource room teacher and significant caregiver as in need of area and perimeter problem-solving skills for everyday use and for more advanced mathematics courses. Three students, Bill, Amber, and Bob, currently receiving a portion of their instruction in a resource room, met criteria for inclusion in the study. Characteristics of these students are presented in Table 1.

All students had IEP goals that involved solving area and perimeter problems. Additionally, Bill's goals included measuring angles, locating main ideas and facts in reading selections, and writing notes for a research paper. Bob's goals included differentiating between positive and negative numbers, creating graphs, using a number line, reading an outline, recording events in order, and increasing reading one grade level. Amber's goals involved staying on task, touching others appropriately, proofreading, recognizing root words, reducing fractions, and differentiating between positive and negative numbers. Despite years of training in the general education classroom and special education resource rooms, all students were

TABLE 1  
Characteristics of Participants

Characteristics	Participants		
	Bill	Amber	Bob
<i>Demographics</i>			
Age (years)	16	13	15
Gender	Male	Female	Male
Grade	9	7	10
Race	Caucasian	African American	Caucasian
Free lunch	Yes	Yes	Yes
Socioeconomic status	Below average	Below average	Below average
<i>Aptitude</i>			
WISC-R Full Scale	90	98	88
<i>Achievement<sup>a</sup></i>			
ITBS computation	1st	1st	5th
WJ: Broad math scores	30th	16th	30th

WISC-R = Wechsler Intelligence Scale for Children-Revised (Wechsler, 1974); WJ = Woodcock-Johnson Psycho-Educational Battery (Woodcock & Johnson, 1999); ITBS = Iowa Test of Basic Skills (Hoover, Hieronymus, Frisbie, & Dunbar, 1993).

<sup>a</sup>Achievement scores are national percentiles.

significantly behind in their math classes. The majority of past instruction had been traditional in nature, emphasizing abstract presentation and solutions to math problems with little or no hands-on or manipulative instruction.

**Setting**

All training in solving area and perimeter problems was conducted on a one-on-one basis in a resource room for students with LD. A teacher certified in special education conducted training on a daily basis for 15–20 minutes.

**Materials**

Materials used in the study included three geoboards (see Figure 1), rubber bands, a 25-foot measuring tape, a pretest in the *Intermediate Geometry: Grade 4* (Fetty, 1999), and a hand-made model doll house consisting of two bedrooms, a bathroom, kitchen, and living room.



FIGURE 1 A sample geoboard perimeter problem.

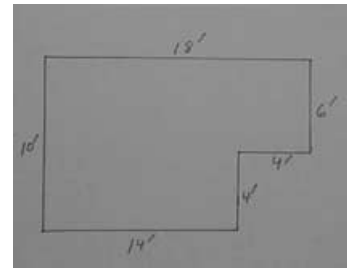


FIGURE 2 A sample teacher-generated perimeter/area problem.

Problems were selected randomly from *Geometry for Primary Grades* (Fitzgibbon, 1999), *Intermediate Geometry* (Fetty, 1999), and *Middle Grade Math: Tools for Success* (Chapin, Illingworth, Landau, Masingila, & McCracken, 1997).

Problems on the geoboard required only single-digit multiplication (area) or single-digit addition (perimeter). This restriction was dictated by the fact that the geoboard consisted of a 9 × 9 array. An example of the set-up of the geoboard just prior to solving an area problem by counting the number of square units is depicted in Figure 2. Abstract problems presented on paper required students to employ double-digit multiplication and double-digit addition with regrouping. Problems presented on paper included figures of pools, tables, desks, and flooring with measurements depicted in inches and/or feet (see Figure 3). All problems presented included only 90° angles because geoboards are not suitable for teaching solutions to problems involving angles other than 90°.

**Experimental Design**

The design employed for this study was a multiple baseline design across people and two behaviors, perimeter and area problem-solving ability (Tawney & Gast, 1984). Baseline measures were obtained by asking students to solve area and perimeter problems presented on the geoboard and on paper.

After baseline data were gathered, Bill was first to begin instruction, while Amber and Bob remained on baseline. Amber’s and Bob’s perimeter or area problem-solving behavior was probed while on baseline. Second,

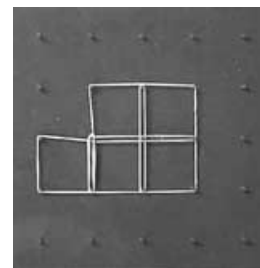


FIGURE 3 A sample geoboard area problem.

when Bill's data indicated improvement in perimeter or area problem solving, Amber began treatment. Third, Bob was exposed to treatment once Amber began acquiring the perimeter- or area-solving skill.

Area and perimeter problem-solving skills were deemed acquired when each student solved correctly, without prompting, 80 percent or more of the problems attempted on each of three consecutive days. Students initially were trained in solving perimeter problems and then in area problems. When students met the 80 percent or greater criterion for solving area and perimeter problems, training ceased, and maintenance checks were performed. The students' teacher performed these checks two times per week in the resource room over the course of three weeks.

The teacher individually asked each child with the aid of the geoboard to calculate the answer to five perimeter and five area problems. In addition, after a two-week Christmas break, students were asked to use paper and pencil to calculate the answers to four area and four perimeter problems. Additionally, students were asked to measure rooms in a model doll house and to convert these scale measures to actual size measures in order to order floor and window coverings.

## Intervention

Modeling, prompting/guided practice, and independent practice in conjunction with manipulative training were employed to teach both perimeter and area problem-solving skills. All students began the intervention learning to solve perimeter problems. Once they mastered this skill they then learned to solve area problems.

### *Perimeter Problem Solving*

The following procedure was used to teach perimeter problem solving. In the first two sessions only, the teacher told the student perimeter meant how far it is around the\_\_\_ (gave examples, such as this room, your yard, etc.) and *modeled* the concept of perimeter to the student by walking all the way around the room. The teacher then *modeled* how to make shapes on the geoboard and allowed the student to create various shapes on the board. In all subsequent sessions, the following activities were initiated. First, the teacher demonstrated on the geoboard that the perimeter of shapes could be determined by counting from nail to nail (the distance from one nail to another nail is one unit). The teacher constructed a square on the geoboard and the student placed a rubber band at the corner of the square as a marker and moved a finger around the square from nail to nail, counting along with the teacher one unit, two unit, and so forth until returning to the rubber band. The student then stated how many units comprised the perimeter. If errors were made in counting and/or in stating the answer, the teacher provided correction and then required the student to recount the units and

answer correctly. Second, the teacher created five random shapes on the geoboard, one at a time, and encouraged the student, as in the prior step, to determine the perimeter. Third, the teacher selected two perimeter problems from one of the math books and demonstrated to the student how to solve the problems using a geoboard. The teacher stated: "We need to represent the figure in the book on the geoboard," then created the figure on the board, and compared the figure on the board to the figure in the book, stating: "Yes, they are the same." The teacher then removed the figure from the board and guided the student in representing another figure on the board, and determining its perimeter as in Step 1. Fourth, the teacher selected two more perimeter problems from the math books and allowed the student to solve them on the geoboard, with prompting as necessary. Fifth, the teacher asked the student to measure a desktop or section of rug or tabletop and then calculate the perimeter. Finally, the teacher selected three problems from the math books requiring single-digit addition and allowed the student to solve these problems on the geoboard with no prompting. A sample perimeter problem is depicted in Figure 1. In addition, the students were asked to solve, via paper and pencil, two problems involving addition with regrouping. A typical problem included a figure of a pool, yard, or room (see Figure 2). These activities provided independent practice and served as the daily test.

### *Area Problem Solving*

The following procedure was used to teach area problem solving. In the first two sessions only, the teacher told the student that area means how much room you have to walk around in\_\_\_ (gave examples, such as this room, your yard, etc.) and *modeled* the concept of area to the student via counting room tiles. In all subsequent sessions, the following activities were initiated. First, the teacher constructed a four-by-four nail square on the geoboard and asked the student to divide the square in as many one-by-one nail squares as possible. Then the student, along with the teacher, counted the number of squares to determine the number of square units (see Figure 3). If errors were made in counting and/or stating the answer the teacher provided feedback: "No, you said\_\_\_ square units. That is incorrect, there are\_\_\_ square units." The teacher then required the student to recount the square units and answer correctly. Second, the teacher created five random shapes on the geoboard one at a time and encouraged the student, with prompting, to determine the area of each shape. Third, the teacher selected two area problems from one of the math books and demonstrated to the student how to solve the problems using a geoboard. The teacher stated: "We need to represent the figure in the book on the geoboard," then created the figure on the board, and compared the figure on the board to the figure in the book, stating: "Yes, they are the same." The teacher removed the figure from the board and then

guided the student in representing another figure on the board and determining its area as in Step 1. Fourth, the teacher selected two more area problems from the math textbooks and allowed the student to solve them on the geoboard with prompting as necessary. A sample area problem is depicted in Figure 3. Fifth, the teacher asked the student to measure a desktop or section of rug or tabletop and then calculate the area. In addition, the students were asked to solve, via paper and pencil, two problems involving multiplication with regrouping. A typical problem included a figure of a pool, yard, or room. (see Figure 2). These activities provided independent practice and served as the daily test.

### Measurement Procedures

The teacher recorded the number of problems the student solved correctly, the type of errors made by the student, the length of time the student required to solve the problems, and the duration of the training session. Problems were scored simply as correct or incorrect. Errors made by the student were recorded as to specific type: (1) representing the problem incorrectly on the geoboard, (2) miscounting the number of units (peg to peg) on the geoboard, (3) miscounting the number of square units on the geoboard, (4) employing the incorrect operation for area or perimeter, (5) computing error, and (6) errors in converting from scale.

### Social Validity

Students were asked to respond verbally to questions about the helpfulness of instruction. In addition, the teacher was asked to comment on the delivery of instruction.

### Treatment Fidelity

Treatment fidelity was evaluated during 25 percent of the sessions. A classroom assistant was given a list of the steps the teacher was to follow when administering treatment and was asked to note whether each step was followed in the same manner for each student. Inspection of these data revealed 100 percent adherence to prescribed method of treatment.

### Interrater Reliability

Reliability of scoring student responses was evaluated by having the assistant observe and score students' solutions to problems during 25 percent of the treatment sessions. There was a 100 percent agreement between the teacher's and assistant's scoring of students' solutions to problems.

## RESULTS

An examination of baseline data presented in Figure 4 reveals that none of the students was able to solve correctly any of the area or perimeter problems presented to them during the baseline conditions. Each time intervention was introduced for area as well as perimeter problem solving, there was a marked increase in problem-solving performance. Bill reached performance criterion (80 percent or greater problems solved correctly on three or more consecutive days) in perimeter problem solving after six days and in area problem solving after five days. Amber required seven days to reach criterion in perimeter problem solving, and five days to attain criterion in area problem solving. Bob became proficient in perimeter problem solving in five days and in area problem solving in five days.

Maintenance checks performed twice per week in each of three consecutive weeks following treatment revealed that all students solved all area and perimeter problems correctly when using both the geoboard and paper and pencil to calculate answers. Immediately following Christmas vacation, Bill, Amber, and Bob were given four area problems involving double-digit multiplication and four perimeter problems involving double-digit regrouping and asked to solve them using only paper and pencil. Their scores were 100 percent, 90 percent, and 90 percent, respectively.

As a test of generalization, the teacher had the students measure room and window sizes in the model dollhouse, convert these measures from scale size to actual size, and determine the appropriate amount of flooring and window treatments to order. The students all successfully used their geoboard skills and paper and pencil problem-solving skills to calculate the sizes required.

As a group, the students made few errors, and those they did commit occurred during initial training sessions and during the generalization exercise. During early sessions, each student made a number of errors in multiplying basic facts and regrouping. Also, they counted the number of pegs rather than the spaces between pegs when initially determining perimeter. Students made no errors when counting square units nor when determining the appropriate procedure to use to calculate area or perimeter. During the generalization exercise, the students at first had difficulty converting from scale size to actual size.

Verbal feedback from students indicated they all liked to "touch/feel" the problems. Initially, Bob thought the geoboard was for "babies," but after one session he indicated that "this was fun and worked for him." Bill remarked that the "book was flat" and that "this," the geoboard, "made problems come alive." Amber noted that "the board allowed her to work longer than the book." The teacher indicated that the procedures were easy to follow, the materials inexpensive, and the students wanted to work with the geoboard each session.

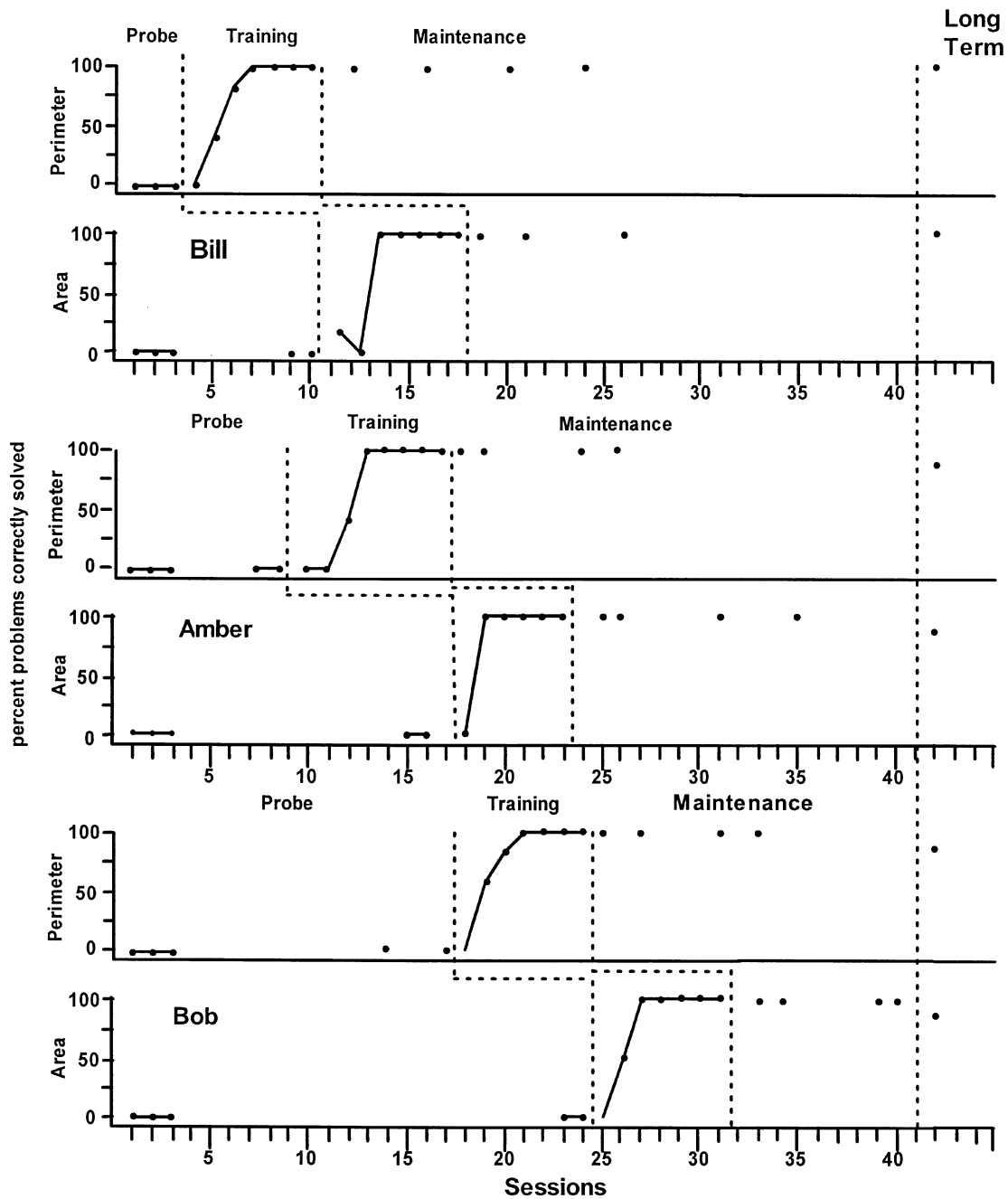


FIGURE 4 The percent of correctly solved perimeter and area problems.

### DISCUSSION

The purpose of this study was to evaluate the effect of manipulative instruction on the acquisition and maintenance of perimeter and area problem-solving skills. Clearly, the treatment resulted in the rapid acquisition and maintenance of basic perimeter and area problem-solving skills. Also, the training appears to have resulted in transfer of skills learned to paper and pencil problem-solving skills.

The results of this study add to the body of literature in several ways. First, they replicate previous findings in the field that indicated manipulative devices were effective in promoting the acquisition and maintenance of various math skills (Funkhouser, 1995; Huntington, 1994; Jordan et al., 1998; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Marsh & Cooke, 1996; Mercer & Miller, 1992; Miller et al., 1998; Miller et al., 1992; Peterson et al., 1988; Witzel, 2001). Second, they extend previous findings by indicating that such instruction

results in long-term retention of skills learned. Third, the present findings suggest, as did one other study (Marsh & Cooke, 1996), that the semiconcrete phase of the CSA sequence may not be a vital component of instruction. Fourth, the findings add to the scant literature related to teaching adolescents with LD more advanced math skills. To date, the present study is the only one that addresses geometry concepts. Finally, the study provides suggested strategies for addressing the NCTM Standards (2000) relating to the use of concrete models, formulas, and the use of measurement in everyday life.

### Educational Implications

Employment of concrete manipulatives in conjunction with modeling, guided practice, and independent practice helped students determine the correct procedures to use when computing the area and perimeter of various figures they encounter in everyday life. Although students maintained these skills over a period of several months, generalization to practical problem solving was accomplished only after students received training in scale conversion. Thus, training in scale conversion prior to testing or training for generalization is recommended. Other implications that educators might consider are: (1) the adaptation of modeling, guided practice, and independent practice to suit individual student learning styles, (2) instruction of students in small groups using peer tutoring, and (3) construction of their own geoboards for a nominal cost.

### Limitations and Future Research

One limitation of this study is that a socially valid test of generalization was not employed. Would the students be able to perform area and perimeter problem-solving skills in their general education classrooms in group settings with limited teacher supervision? Another limitation involves the type of problems given; none contained angles other than  $90^\circ$  because geoboard instruction does not allow for such representations. However, in real life, angles must be taken into consideration when solving area and perimeter problems. The success the students demonstrated could be attributed to their teacher's level of enthusiasm and skill in delivering instruction, the concrete depiction of the concepts of area and perimeter that, according to one student, "made the book come alive," the amount of modeling and guided or independent practice provided, or a combination of these factors. Also, the findings are limited due to the inability to generalize to a larger population based on the employment of such a small sample size.

Future researchers need to examine means to effectively promote generalization of area and perimeter problem-solving skills and ways to incorporate non- $90^\circ$  angles into manipulative instruction. In addition, studies employing group designs to compare concrete

to semiconcrete to abstract instruction are warranted. Also, researchers need to design studies to determine the contributions that various instructional strategies, individually and in combination, have on learning math skills.

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