

Solve It! Strategy Instruction to Improve Mathematical Problem Solving

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Solve It! is a research-based instructional program for students who have difficulty solving mathematical word problems. Many of these students require particular help in understanding and using cognitive processes and self-regulation strategies that underlie effective and efficient problem solving. This article underscores the need to improve students' mathematical problem solving, describes the theoretical framework for the instructional program, reviews the research on *Solve It!*, presents an overview of the program, and discusses several practical issues related to its implementation. This mathematical problem-solving program has been successful for students with mathematical learning disabilities and, therefore, can be used in inclusive, general education classrooms as well as special education classes.

Middle school mathematics teacher Ms. Morgan asked her diverse group of students why they need to know how to solve mathematical problems like the following: Caroline owns a dog kennel. She usually has 15 dogs to care for every week. Each dog eats about 10 lb. of food per week. She pays \$1.60 per pound for the food. How much does Caroline pay to feed 15 dogs each week? Ms. Morgan's students replied that they need mathematical problem solving for "careers and future jobs, work like accountants in banks, NASA and space exploration, doing taxes, home remodeling and construction, measuring things around the house, paying bills, and getting change." There was no doubt that these middle school students understood the importance of mathematical problem solving as a practical skill that is useful in daily life and essential in the workplace. Despite its real-life relevance, however, students continue to have difficulty learning how to solve mathematical problems. As Ms. Morgan pointed out, "Problem solving is difficult because it involves critical thinking. These are the skills that students need in their everyday lives. Therefore, we need to teach problem solving so that students will be prepared."

This article describes *Solve It!*, a research-based instructional program for teaching students who have difficulty in mathematics how to solve mathematical word problems. Many students need particular help in acquiring and applying the cognitive and metacognitive processes and strategies that underlie effective and efficient problem solving. *Solve It!* has been particularly effective for students with mathematical learning disabilities and, therefore, can be used successfully in inclusive and general education classrooms as well as spe-

cial education classes (Montague, 1997a). A transcription of an actual *Solve It!* lesson taught by a general education middle school mathematics teacher appears later in the article.

A LOOK AT SOLVE IT!

Curriculum reform in mathematics is based on the premise that mathematics is a way of thinking that relies on the ability to understand and represent problem situations; organize and classify relevant information; draw on appropriate mathematical knowledge and know where, when, how, and why to apply that knowledge; and explain the concepts underlying problem solutions and why certain procedures are used (Mathematical Sciences Education Board, 1989; National Council of Teachers of Mathematics [NCTM], 1989, 1991). The traditional and conventional mathematics curriculum is based on rote acquisition of declarative and procedural knowledge with little regard for developing conceptual and strategic knowledge in students. The NCTM Standards are more relevant to real-world mathematical experiences in that they advocate using problem solving as the centerpiece for mathematics instruction. This new curricular concept emphasizes teaching students to use the mathematics they are learning in their daily lives (Davis & Maher, 1996; Montague, 1996b).

As the academic curricula change to meet the needs of a changing society, so must instruction change. The traditional model, in which the teacher demonstrates the procedures needed to solve a problem and then provides time for drill and practice, does not meet the educational needs of most learners, especially students with learning and behavioral problems (Miller, Butler, & Lee, 1998). The typical textbook

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approach to solving mathematical word problems—that is, read the problem, decide what to do, solve it, and check your answer—does not provide students with the necessary cognitive tools for problem solving. The critical question remains: How does an individual decide what to do and how do we teach this process?

Solve It! provides the extra instructional assistance for teaching students how to decide what to do. Students learn how to understand the mathematical problems, analyze the information presented, develop logical plans to solve problems, and evaluate their solutions. Students are taught how to read the problem for understanding, paraphrase by putting the problem into their own words, visualize the problem by drawing a picture or making a mental image, set up a plan for solving the problem, estimate the answer, and compute and verify the solution. Students are also taught self-regulation strategies needed for effective problem solving, which include self-instruction, self-questioning, and self-monitoring. These strategies help problem solvers gain access to strategic knowledge, guide learners as they apply strategies, and regulate their use of strategies and their overall performance as they solve problems. They can be used overtly (talking out loud or whispering to oneself) or covertly (thinking to oneself). Self-instruction implies telling oneself what to do before and while performing actions. Self-questioning means asking oneself questions while engaged in an activity to stay on task, regulate performance, and verify accuracy. Self-monitoring requires the problem solver to make certain that everything is done correctly throughout the problem-solving process. Figure 1 lists the cognitive processes

and corresponding self-regulation strategies that are basic to solving mathematical problems.

Solve It! also provides teachers with a set of proven instructional techniques that help students actively acquire and effectively apply the cognitive processes and self-regulation strategies (Montague, 1992, 1996a; Montague, Applegate, & Marquard, 1993). The four major instructional techniques include (a) problem-solving assessment, (b) explicit instruction of problem-solving processes and strategies, (c) process modeling, and (d) performance feedback.

Problem-Solving Assessment

Initial assessment and ongoing monitoring to determine individual student progress are both critical for a variety of reasons. At the outset, it is important not only to measure student performance in solving mathematical problems, but also to ascertain each student's strategic knowledge and use of strategies. As students become better problem solvers, both performance and strategic knowledge and use are measured. *Solve It!* relies on assessment procedures that are student centered, process oriented, and directly relevant to the instructional program. Understanding a student's knowledge base, skill level, learning style, information processing, strategic activity, attitude, and motivation for learning mathematics helps teachers plan instruction that is effective, efficient, and geared to the attributes of individual students. By assessing these attributes prior to the start of the instructional program, the teacher is able to make judgments about both individual and group instructional needs. There are two types of informal assessment techniques built into this program that are used as pretests and to document student progress.

The first type of assessment consists simply of criterion-referenced tests of 10 one-, two-, and three-step mathematical word problems. Students are given a pretest to determine baseline performance and then are tested periodically throughout the instructional program to ensure that progress is continual and that they reach mastery. These periodic progress checks inform the teacher about individual student performance. In this way, teachers can adjust the program based on individual student needs.

The second type of assessment measures student knowledge and use of mathematical problem-solving strategies. The Mathematical Problem-Solving Assessment—Short Form (MPSA-SF; Montague, 1992) is an informal assessment tool that was adapted from a longer version (Montague, Bos, & Doucette, 1991). The original MPSA was developed as a research tool to assess a variety of important cognitive and noncognitive factors associated with mathematical problem solving. To make the instrument more practical and useful to teachers, the MPSA-SF was developed. The MPSA-SF is to be used as an informal diagnostic-prescriptive tool to identify individual students' strengths and weaknesses in mathematical problem solving (see Montague, 1996a, for a detailed explanation of the MPSA-SF).

At the beginning of the instructional program, students usually take about 45 min to solve 10 word problems. As they become better problem solvers, they become more efficient and will take less time. Pretesting provides a performance

Read (for understanding)	
Say:	Read the problem. If I don't understand, read it again.
Ask:	Have I read and understood the problem?
Check:	For understanding as I solve the problem.
Paraphrase (your own words)	
Say:	Underline the important information. Put the problem in my own words.
Ask:	Have I underlined the important information? What is the question? What am I looking for?
Check:	That the information goes with the question.
Visualize (a picture of a diagram)	
Say:	Make a drawing or diagram.
Ask:	Does the picture fit the problem?
Check:	The picture against the problem information.
Hypothesize (a plan to solve the problem)	
Say:	Decide how many steps and operations are needed. Write the operation symbols (+ - x ÷).
Ask:	If I do —, what will I get? If I do —, then what do I need to do next? How many steps are needed?
Check:	That the plan makes sense.
Estimate (predict the answer)	
Say:	Round the numbers, do the problem in my head, and write the estimate.
Ask:	Did I round up and down? Did I write the estimate?
Check:	That I used the important information.
Compute (do the arithmetic)	
Say:	Do the operations in the right order.
Ask:	How does my answer compare with my estimate? Does my answer make sense? Are the decimals or money signs in the right places?
Check:	That all the operations were done in the right order.
Check (make sure everything is right)	
Say:	Check the computation.
Ask:	Have I checked every step? Have I checked the computation? Is my answer right?
Check:	That everything is right. If not, go back. Then ask for help if I need it.

FIGURE 1 Cognitive processes and self-regulation strategies for mathematical problem-solving instruction.

baseline for each student receiving mathematical problem-solving instruction. Progress checks are conducted periodically throughout the instructional program, and performance can be compared to baseline performance. Progress checks should be administered on a regular schedule to keep a continuous record of student performance. The ultimate goal of *Solve It!* is to achieve a criterion of at least 7 out of 10 problems correct on four consecutive word problem tests. Student progress graphs can be scanned quickly to determine whether students are making constant progress and are moving toward mastery. Figure 2 shows a student's *Solve It!* graph. It is important to engage students actively in assessing their own progress by having them chart their performance on the progress check graphs, which are kept in portfolios with the progress checks. In this way, the student's improvement over time is demonstrated and the success of the instructional program is established.

Explicit Instruction of Problem-Solving Processes and Strategies

Explicit instruction in mathematical problem solving requires highly structured and organized lessons, appropriate cues and prompts, guided and distributed practice, immediate and corrective feedback on learner performance, positive reinforcement, overlearning, and mastery. Explicit instruction is flexible because it allows teachers to adapt the teaching routine to fit their own style and tailor instruction to consider the strengths and weaknesses of their students. However, *Solve It!* has been found to be most effective if teachers adhere generally to the instructional model and follow the teaching routine. The core instructional procedures associated with explicit instruction promote active learning, acquisition of mathematical problem-solving processes and strategies within a reasonable time period, and effective and efficient application when solving mathematical problems. Each lesson is scripted to ensure that all of the instructional procedures associated with explicit instruction are incorpo-

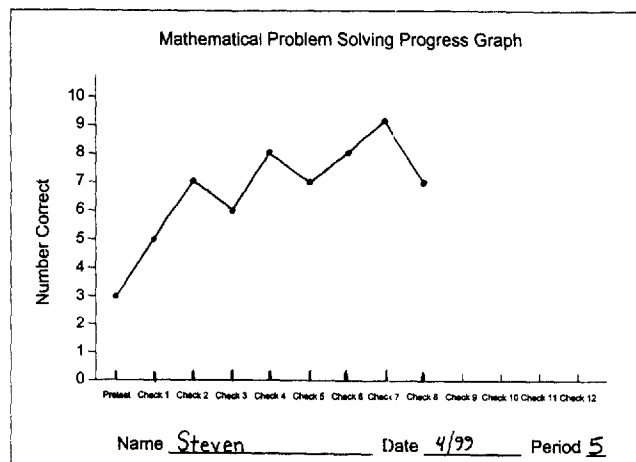


FIGURE 2 Mathematical problem-solving progress graph.

rated. This makes the teaching routine easy to learn and ensures that the cognitive processes and self-regulation strategies are effectively integrated within each lesson, and that students understand them and are provided with frequent practice in applying them with a variety of mathematical problems. Inherent within the instructional procedure is a teaching process that sets the stage for learning; plans for relevant exemplars; and develops an understanding of skills through reinforcement, repetition, and mastery learning.

Solve It! teaches mathematical problem solving through a guided discussion technique using a specific problem-solving routine. Students are actively engaged in the learning process through an initial discussion of the importance of mathematical problem solving, their individual performance on a pretest, and performance goals that they set for themselves. After learning the processes and strategies and observing several demonstrations of effective problem solving, students are engaged in numerous practice sessions where they apply what they have learned. From the outset, students usually are enthused about the program because they experience immediate success. As they are guided through scripted lessons and application sessions, they are given practice in verbalizing the cognitive processes and self-regulation strategies they use. They also are given practice in justifying their approach to the problem and responding to the teacher and to one another. If students are reluctant to participate, the teacher should model answers to encourage them and help them verbalize their thoughts. The teacher must make it clear that participation through verbalization and demonstration is necessary and that this will help students to become good problem solvers. Students need to understand the benefits of becoming a good problem solver.

Process Modeling

Process modeling is thinking aloud while demonstrating a cognitive activity. The teacher and then the students use process modeling to help them apply the problem-solving processes and strategies. This instructional strategy stresses learning by imitation and provides students with the opportunity to observe and hear how to solve mathematical problems. As the model, the teacher shows students how to say everything they are thinking and doing as they solve the mathematical problems. The teacher shows students not only what to do but also what not to do. Modeling of correct behaviors allows students to observe appropriate and successful application of the processes and strategies. Modeling of incorrect behaviors and responses allows students to observe what it means to locate and correct errors as they solve problems. This technique will develop self-monitoring skills in students. After students learn the modeling routine, then they become the models for their classmates.

The self-regulation strategies that students have learned facilitate thinking aloud by providing self-instructions and questions to ask. At first, students may be shy and reluctant to think out loud while they are solving problems. But with practice, they become quite adept. Initially, students will need support, encouragement, and reinforcement for thinking aloud as they demonstrate problem solving for the other stu-

dents. As they become more proficient problem solvers, they will progress to using covert self-regulation strategies when they solve problems independently. During practice sessions when students are solving problems and applying the problem-solving processes and strategies, students can take turns being the teacher. They may use an overhead projector or the chalkboard and model the problem-solving routine for the other students. Students in the class may ask questions to develop students' ability to justify their approach to the problem. The teacher may take the role of a student while the class leads and guides her through the problem-solving process.

Performance Feedback

Students are always given specific feedback regarding their performance and responses as they learn and apply the problem-solving processes and strategies. Performance during practice sessions and periodic progress checks is also carefully analyzed. The teacher is responsible for helping students learn to use the processes and strategies and become more efficient as well as effective problem solvers and appraise, critique, and monitor their own performance. Because feedback is critical to making students aware of what they are doing, how they interact, and how they can improve their problem solving, every student must be actively involved in the process. This involvement should be encouraged throughout the instructional and practice sessions. Students should be praised by their peers and the teacher for solving problems correctly and improving on the periodic progress checks. The concept of achieving a "personal best" is emphasized. The teacher always models the use of labeled praise by directing the feedback toward the appropriate student. It is important to involve all students by calling directly on those who are most reluctant to participate and then praising them for giving appropriate feedback. If students are unsure of what to say or how to give feedback, the teacher may need to interject with prompts such as "You could say, 'Allen, your diagram is really clear and organizes the information in the problem for me.' Now you tell Allen."

Reinforcement is essential for students who are learning and practicing problem solving. Students need to know exactly which behaviors and responses are being praised so that they can be repeated. They should also be provided with opportunities to practice giving and receiving positive feedback and praise. Reinforcement shows them that they are successful and can become better problem solvers. Praise must not be exaggerated; rather, it must reflect an honest appraisal of students' responses. If students are going to maintain and generalize problem-solving skills to other settings and situations, they must develop confidence in their ability to solve problems. Positive reinforcement serves to inform students that they are performing well and are making progress as they become proficient problem solvers. Peer reinforcement for participating in practice sessions is an important part of the program. As students learn how to solve problems and then practice mathematical problem solving, they will become more independent, less fearful of being wrong, and surer of themselves when they solve problems in or outside class. They need feedback on what and how they are doing so that

solving mathematical problems becomes an enjoyable and nonthreatening activity. The ultimate goal is to have students recognize that they have done well and praise themselves for doing well.

EFFECTIVENESS OF SOLVE IT!

Solve It! has been tested in three different studies with a total of 84 students with learning disabilities (LD). Several descriptive studies were conducted to first determine the characteristics of problem solvers (Montague, 1991, 1997b; Montague & Applegate, 1993a, 1993b). These studies, which guided the further development and refinement of the instructional program, suggested that the most salient characteristic of students with LD was their inability to represent problems. Problem representation processes include paraphrasing, visualizing, and hypothesizing or making a plan to solve a problem.

The ability to paraphrase a mathematical problem means being able to translate the linguistic information in the problem by rephrasing or restating the problem. Putting the problem into one's own words without changing the meaning of the problem is the test of good paraphrasing. Students should be taught how to paraphrase parts of problems and then "tell the story" to others in a way that conveys the meaning of the problem.

Good problem solvers also use visualization to help them process the linguistic and numerical information in a mathematical problem and form internal representations in memory. They do this by either drawing a representation on paper or making a mental image of the problem. These images can be geometric representations, diagrams, tables, figures, or some other type of graphic or pictorial display. Some problem solvers imagine the "story" and may actually see themselves and others as characters in the story.

Good problem solvers then develop a solution hypothesis, which is directly tied to their comprehension of the problem and integration of the problem information. These representation strategies assist problem solvers in deciding on a solution path that includes establishing a goal, looking toward the outcome, and setting up a plan to solve the problem. This entails deciding on the number of operations that are needed to solve the problem, selecting and ordering the operations, and then transforming the information into correct equations and algorithms.

The first intervention study focused on 6 senior high school students with LD who were taught individually (Montague & Bos, 1986). The instructional program was then refined and another study was conducted with 6 students with LD in Grades 6–8 (Montague, 1992). Finally, a large group study was completed with 72 students with LD in Grades 7 and 8 who were taught in small to midsize groups (Montague, Applegate, & Marquard, 1993). Although all students improved substantially in the individualized instruction studies, the 2 sixth-grade students did not reach the criterion for mastery, which was set at 7 correct problems out of a possible 10 on four consecutive mathematical problem-solving tests. This finding suggested that the content of the instructional program might have been too developmentally advanced for these young students. Results of a previous study had also

raised the issue of developmental appropriateness of the program (Montague & Applegate, 1993a). Therefore, the group intervention study included only students in Grades 7 and 8. Results indicated that, following instruction, students with LD approximated the performance of the comparison group, average-achieving students who were not given instruction but had demonstrated problem-solving ability. Generally, across studies, students maintained strategy use and problem-solving performance for several weeks following instruction, after which performance declined. However, following a booster session consisting of a day of review and another for practice, students demonstrated significant improvement. In these intervention studies, scripted lessons were sequenced to ensure that students learned and understood the cognitive processes and self-regulation strategies associated with effective problem solving.

The lessons for *Solve It!* were patterned after the research intervention. For *Solve It!*, each lesson has an overall instructional goal and behavioral objectives that reflect the content of the lesson. Lists of materials for each lesson indicate which instructional charts, practice problems, activities, and cue cards are needed. Explicit instructional cues help the teacher pace the lesson by indicating which procedures to use and when to use them. The lesson script is divided into several steps. Following mastery of the comprehensive problem-solving routine during the first three lessons (see Figure 1), formats for practice sessions are provided to ensure that students have ample opportunity to practice and reach the goal of improved performance (at least 70% correct on progress checks of 10 one-, two-, and three-step word problems for four consecutive checks). Lessons and practice sessions are structured to reinforce previous learning. Review is built into every lesson to reinforce acquired processes and strategies and to help students internalize learning. Scripted lessons for the booster sessions are also included, which are needed to help students maintain strategy use and improved performance over time.

Lesson One starts with an overview of *Solve It!* First, the teacher guides a discussion among students about mathematical problem solving in general and why it is important to be a good problem solver. Then, the program is described for the students, and the various cognitive processes that they will learn are presented on a master class chart. Students practice verbalizing the processes and strategies by reading through the charts individually and as a group using choral reading techniques. The teacher then demonstrates how to use the comprehensive strategy to solve typical mathematical word problems. Students have an opportunity to ask questions, and the lesson concludes with distribution of the study booklets.

At the beginning of Lesson Two, volunteer students are tested for mastery of the seven cognitive processes. The teacher provides corrective and positive feedback and checks off the names of students who are able to recite from memory the names and descriptions of the processes. Then the entire group practices recitation. Individual students then take turns reciting the processes from memory. Students are cued using the acronym RPV-HECC and the master class charts that are posted on the walls of the classroom. Following group and individual recitation of the processes, the teacher reviews the metacognitive strategies with the students and leads the group

as they recite the processes and strategies using the master class charts. The teacher then models problem solving for the students. An example of process modeling appears later in this article. On subsequent practice problems, the students may be called on to verbalize the processes and strategies as the teacher works through the problem. If there is time, students practice reciting the processes and strategies.

At the start of Lessons Three, Four, and Five, students are tested for mastery of the processes. Again, names of students who have reached 100% criterion for recitation of the processes are checked off by the teacher. The teacher leads the group as they recite all the processes and the SAY, ASK, CHECK strategies. The students then can solve a practice problem individually at their seats. They are instructed to think aloud and verbalize the processes and strategies as they solve the problem. The teacher or a student models the correct solution. Later, students can be paired for solving problems. Partners take turns telling one another what to do. Students are given only one problem at a time to solve during these initial lessons. After completing each problem, the teacher or a student models the correct solution. Students and the teacher can assist the problem solvers by verbalizing the processes and strategies as they work through the problem. Rehearsal activities conclude the lesson.

The criteria for moving to Lesson Six include the following: (a) all students in the group meet the mastery criterion (100%) for recitation of the cognitive processes from memory (Figure 1); (b) students understand and are able to use the SAY, ASK, CHECK strategies; and (c) students can work through the math problems with relative comfort and confidence. For students who do not meet these criteria, the routine for Lessons Three through Five should be repeated.

In Lesson Six, students are given the first practice set of 10 problems to solve. They are cued to use the strategy, consult the master class charts or their booklets, and think aloud. Following completion of each problem, either the teacher or a student can model the correct solution. Lesson Seven requires the students to solve all 10 problems prior to modeling the correct solutions for the problems. They should be encouraged to ask questions and discuss the solutions.

Lesson Eight is the first Progress Check (test of 10 problems). Students may "grade" their own papers or exchange papers for "grading." They need to understand that they are in the process of becoming good problem solvers and that they will become better as time goes on. Students plot their "grade" on their performance graph and then model the solutions for their classmates.

A LOOK IN MS. MORGAN'S CLASSROOM

Ms. Morgan has 21 students in her sixth period, eighth-grade general mathematics class. The students were placed in the class because they received a C or less in their first semester mathematics class. Five of the students have identified learning disabilities and receive resource room support. All of the students have difficulty solving mathematical word problems. This is the 4th day of *Solve It!* instruction for these students. During the first three lessons,

students engaged in a variety of group recitation activities and observed Ms. Morgan solving problems. At the beginning of Lesson Four, all of the students had reached 100% criterion in recitation of the cognitive processes from memory. Ms. Morgan also observed that they seemed comfortable with the SAY, ASK, CHECK procedure and seldom needed to consult the wall charts. Ms. Morgan had modeled the problem-solving process for the students several times during the previous lessons. Also, individual students had “guided” her through the process as she solved problems. Ms. Morgan plans to begin Lesson Four by modeling a solution once again before having the students solve problems on their own. The following is a transcription of Ms. Morgan as she models mathematical problem solving.

First, she presents the problem on an overhead projector and says, “Now watch me say everything I am thinking and doing as I solve this problem: On Monday, Alex bought 14 gallons of paint for the house. On Tuesday, he bought 12 more gallons. After painting the house, he returned 3 unopened gallons to the store. How much paint was used?”

First, I am going to *read* the problem for understanding.

SAY: Read the problem. Okay, I will do that. (Ms. Morgan reads the problem.) If I don’t understand it, I will read it again. Hm, I think I need to read it again. (She reads the problem again.)

ASK: Have I read and understood the problem? I think so.

CHECK: For understanding as I solve the problem.

Next, I am going to *paraphrase* by putting the problem into my own words.

SAY: Put the problem into my own words. Alex bought 14 gallons and then he bought another 12 gallons of paint and took back 3 gallons. Underline the important information. I will underline “bought 14 gallons” and “bought 12 more” and “returned 3.”

ASK: Have I underlined the important information? Let’s see, yes I did. What is the question? The question is how much paint was used? What am I looking for? I am looking for the number of gallons Alex used to paint the house.

CHECK: That the information goes with the question. I have the number of gallons he bought and the number he returned. I need to find the total number of gallons he used.

Then I will *visualize* by making a drawing or a diagram.

SAY: Make a drawing or a diagram. Hm, I will draw a bucket and label it “14” and draw another and label it “12” and draw a big circle around them to tell myself that it is the total.

Then, I will draw a third bucket and label it “3” and put a minus sign before it.

ASK: Does the picture fit the problem? Yes, I believe it does tell the story.

CHECK: The picture against the problem information. Let me check the numbers, 14, 12, 3, and it makes sense.

Now I will *hypothesize* by making a plan to solve the problem.

SAY: Decide how many steps and operations are needed. Let me see. First I need to get the total gallons of paint he bought on two different days. Then I need to subtract the leftover paint to see how much he actually used. Okay, add and then subtract, two steps. Now I will write the operation symbols for add and subtract.

ASK: If I (add the two amounts he bought and then subtract the gallons returned), I will get (the number of gallons used). How many steps are needed? Two steps.

CHECK: That the plan makes sense. If not, ask for help. It makes sense.

Next I need to *estimate* by predicting the answer.

SAY: Round the numbers, do the problem in my head, and write the estimate. Round 14 up to 15 and 12 down to 10, add 15 and 10 to get 25 and then take away 3 rounded to 5 for an estimated answer of 20.

ASK: Did I round up and down? Yes, I did. Did I write the estimate? Yes.

CHECK: That I used all the important information. Two steps. Okay.

Now I *compute* by doing the arithmetic.

SAY: Do the operations in the right order. Okay, first add 14 plus 12 equals 26. Then subtract 3 from 26, equals 23.

ASK: How does my answer compare with my estimate? Hm, 20 is the estimate, 23 is the answer. Very close. Does my answer make sense? Yes, he used 23 gallons of paint for the house. Are the decimals or money signs in the right places? None is needed in this problem.

CHECK: That all the operations were done in the right order. Add, subtract. Yes, they were.

Okay, now I really get to *check* to see if the answer is correct.

SAY: Check the computation. Let’s see. I will go backwards. Twenty-three plus 3 equals 26 minus 12 equals 14, which is what he started with.

ASK: Have I checked every step? Yes, the operations chosen were correct, and the computation was correct. Is my answer right? Yes.

CHECK: That everything is right. I have checked everything. If not, go back. Then ask for help if I need it. I do not need help. The answer is right.

Then, Ms. Morgan answers questions the students have. Finally, she gives the students a problem to solve independently. They are instructed to say everything they are thinking and doing and to use the processes and strategies. A student is then selected to model the solution just as Ms. Morgan did.

PRACTICAL ISSUES RELATED TO PROGRAM IMPLEMENTATION

When asked about the practical side of effectively implementing *Solve It!* in her mathematics classes, Ms. Morgan voiced the following concerns:

- Finding the time for assessing students individually could present a problem. Unless a resource teacher or an instructional aide can assist teachers, there simply may not be enough time to administer and score informal instruments like the MPSA-SF with large groups of students.

- Understanding students' strengths and weaknesses in mathematics and then tailoring instruction to meet their individual needs may not be feasible given the large numbers of students enrolled in most middle school teachers' content classes (class size can range from 25 to 40 students, and teachers usually teach at least five classes).

- Not all students in the general education mathematics class may need explicit instruction in mathematical problem solving. Some students have acquired effective problem-solving strategies and use them appropriately. Therefore, identifying the students who need instruction and then grouping for instruction based on the various levels in the class can be a real challenge for a math teacher.

- Changing the thinking habits and improving the problem-solving strategies of students with reading and other learning difficulties seems an unrealistic goal for many teachers. Teachers often feel unprepared to teach students who are in special programs.

- Scheduling time to talk with the resource teacher for students in special education can be difficult given the many obligations of the typical middle school classroom teacher. Additionally, teachers often do not have the time or the inclination to coordinate what is taught in the resource room with instruction in the general education mathematics class. Communication between teachers sometimes can be difficult.

- Teachers often lack the skills and teaching style to implement a program like *Solve It!* successfully. Because the program is intense and highly interactive, teachers may need in-service training to learn the different instructional strategies such as cognitive modeling and verbal rehearsal that are needed for implementing the program.

- Teachers should become familiar with the research that supports strategy instruction and its components as well as the instructional procedures. To illustrate, visual imagery is a recognized mnemonic strategy that enables problem solvers to make mental representations of problems. Students with mathematical problems need instruction in strategies to facilitate problem representation. Teachers who are knowledgeable about the research underlying effective instruction will be able to justify the instructional time spent on *Solve It!* in their classes. They will also be able to explain how the program complements and builds on the mathematics curriculum.

CONCLUSION

Solve It! provides teachers with a research-based program that makes mathematical problem solving easy to teach, and it provides students with a problem-solving routine that makes

mathematical problem solving easy to learn. We know that as students become more effective problem solvers, they also become more efficient and take less time solving ordinary problems. Like good problem solvers, they tend to persevere on more difficult and novel problems. Despite perseverance, however, students often are unsuccessful on novel, complex problems. Therefore, we need to provide problem-solving instruction that will give students the tools to solve authentic, complex mathematical problems they encounter in everyday life. As they apply what they have learned about problem solving, practice solving mathematical problems in a variety of contexts, and document their progress, their confidence as problem solvers will strengthen. According to Ms. Morgan, "*Solve It!* gets students excited about problem solving and, as they become better at it, they begin to like the challenge and, the best part, they actually begin to enjoy mathematical problem solving."

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